2.4

# We're Shipping Out!

## Solving and Graphing Compound Inequalities

### LEARNING GOALS

In this lesson, you will:

- Write simple and compound inequalities.
- Graph compound inequalities.
- Solve compound inequalities.

### KEY TERMS

- compound inequality
- solution of a compound inequality
- conjunction
- disjunction

How many different ways do you think water exists? You may instantly think of water in a liquid state like you see in raindrops, or in lakes, ponds, or oceans. However, you probably also know that water can be a solid like hail, or ice cubes; or as a gas as in the humidity you may feel on a hot summer day, or the steam you see. What factors do you think play a role in the way water exists? Can you think of other things that can take the form of a solid, liquid, *and* gas?

## PROBLEM 1 GoodSportsBuys.com

GoodSportsBuys.com is an online store that offers discounts on sports equipment to high school athletes. When customers buy items from the site, they must pay the cost of the items as well as a shipping fee. At GoodSportsBuys.com, a shipping fee is added to each order based on the total cost of all the items purchased. This table provides the shipping fee categories for GoodSportsBuys.com.

2

| Total Cost of Items | Shipping Fee |
|---|---|
| $0.01 up to and including $20 | $6.50 |
| More than $20 up to and including $50 | $9.00 |
| Between $50 and $75 | $11.00 |
| From $75 up to, but not including, $100 | $12.25 |
| $100 or more | $13.10 |

1. What is the least amount a customer can spend on items and pay $6.50 for shipping?

2. What is the greatest amount a customer can spend on items and pay $6.50 for shipping?

3. What is the shipping fee if Sarah spends exactly $75.00 on items? Explain your reasoning.

4. Harvey says he will spend $13.10 on shipping fees if he spends exactly $100 on items. Is he correct? Explain your reasoning.

5. Consider the table of shipping costs to complete each statement using the phrase "greater than," "less than," "greater than or equal to," or "less than or equal to."

   a. You will pay $6.50 in shipping fees if you spend:

   b. You will pay $9.00 in shipping fees if you spend:

   c. You will pay $11.00 in shipping fees if you spend:

   d. You will pay $12.25 in shipping fees if you spend:

   e. You will pay $13.10 in shipping fees if you spend:

A **compound inequality** is an inequality that is formed by the union, "or," or the intersection, "and," of two simple inequalities.

6. You can use inequalities to represent the various shipping fee categories at GoodSportsBuys.com. If you let $x$ represent the total cost of items purchased, you can write an inequality to represent each shipping fee category. Complete each inequality using an inequality symbol.

   a. $6.50 shipping fees: $x$ ☐ $0.01 and $x$ ☐ $20

   b. $9.00 shipping fees: $x$ ☐ $20 and $x$ ☐ $50

   c. $11.00 shipping fees: $x$ ☐ $50 and $x$ ☐ $75

   d. $12.25 shipping fees: $x$ ☐ $75 and $x$ ☐ $100

   e. $13.10 shipping fees: $x$ ☐ $100

7. Identify the inequalities in Question 6 that are compound inequalities.

Let's consider two examples of compound inequalities.

## 2

$x > 2$ and $x \leq 7$

This inequality is read as "all numbers greater than 2 and less than or equal to 7." This inequality can also be written in the compact form of $2 < x \leq 7$.

$x \leq -4$ or $x > 2$

This inequality is read as "all numbers less than or equal to $-4$ *or* greater than 2."

Only compound inequalities containing "and" can be written in compact form.

**8.** Write the compound inequalities from Question 6 using the compact form.

**a.** \$6.50 shipping fees: _____

**b.** \$9.00 shipping fees: _____

**c.** \$11.00 shipping fees: _____

**d.** \$12.25 shipping fees: _____

## PROBLEM 2 More Than One Solution

1. Water becomes non-liquid when it is 32°F or below, or when it is at least 212°F.
   a. Represent this information on a number line.

2

   b. Write a compound inequality to represent the same information. Define your variable.

2. Luke and Logan play for the same baseball team. They practice at the Lions Park baseball field. Luke lives 3 miles from the field, and Logan lives 2 miles from the field.
   a. First, plot a point to represent the location of the Lions Park baseball field.
   b. Next, use your point that represents Lions Park, and draw a circle to represent all the possible places Luke could live.
   c. Finally, use your point that represents Lions Park, and draw another circle to represent all the possible places Logan could live.

   d. What is the shortest distance, $d$, that could separate their homes?

   e. What is the longest distance, $d$, that could separate their homes?

f. Write a compound inequality to represent all the possible distances that could separate their homes.

g. Represent the solution on a number line.

2

3. Jodi bought a new car with a 14-gallon gas tank. Around town she is able to drive 336 miles on one tank of gas. On her first trip traveling on highways, she drove 448 miles on one tank of gas. What is her average miles per gallon around town? What is her average miles per gallon on highways?

a. Write a compound inequality that represents how many miles Jodi can drive on a tank of gas. Let $m$ represents the number of miles per gallon of gas.

b. Rewrite the compound inequality as two simple inequalities separated by either "and" or "or."

c. Solve each simple inequality.

d. Go back to the compound inequality you wrote in Question 3, part (a). How can you solve the compound inequality without rewriting it as two simple inequalities? Solve the compound inequality.

e. Compare the solution you calculated in Question 3, part (c) with the solution you calculated in Question 3, part (d). What do you notice?

f. Explain your solution in terms of the problem situation.

**g.** Represent the solution on a number line. Describe the shaded region in terms of the problem situation.

## PROBLEM 3 Solving Compound Inequalities

2

Remember, a compound inequality is an inequality that is formed by the union, "or," or the intersection, "and," of two simple inequalities.

The **solution of a compound inequality** in the form $a < x < b$, where $a$ and $b$ are any real numbers, is the part or parts of the solutions that satisfy both of the inequalities. This type of compound inequality is called a **conjunction**. The solution of a compound inequality in the form $x < a$ or $x > b$, where $a$ and $b$ are any real numbers, is the part or parts of the solution that satisfy either inequality. This type of compound inequality is called a **disjunction**.

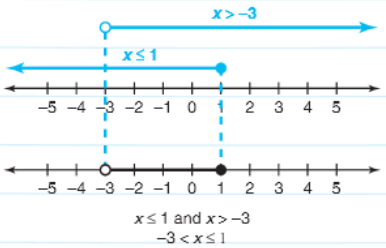
**1.** Classify each solution to all the questions in Problem 2 as either a conjunction or disjunction.

Let's consider two examples for representing the solution of a compound inequality on a number line.

The compound inequality shown involves "and" and is a conjunction.

$x \leq 1$ and $x > -3$

Represent each part above the number line.




$x \leq 1$ and $x > -3$
$-3 < x \leq 1$

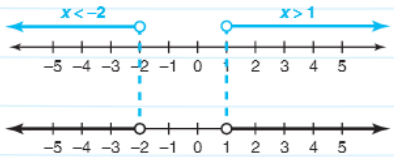
The solution is the region that satisfies both inequalities. Graphically, the solution is the overlapping, or the intersection, of the separate inequalities.

2

The compound inequality shown involves "or" and is a disjunction.

$x < -2$ or $x > 1$

Represent each part above the number line.

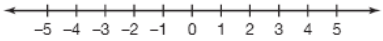



$x < -2$ or $x > 1$

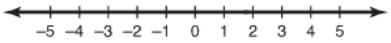
The solution is the region that satisfies either inequality. Graphically, the solution is the union, or all the regions, of the separate inequalities.

**2.** Consider the two worked examples in a different way.

**a.** If the compound inequality in the first worked example was changed to the disjunction, $x \leq 1$ or $x > -3$, how would the solution set change? Explain your reasoning.

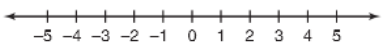



**b.** If the compound inequality in the second worked example was changed to the conjunction, $x < -2$ or $x > 1$, how would the solution set change? Explain your reasoning.

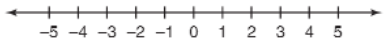



**3.** Represent the solution to each compound inequality on the number line shown. Then, write the final solution that represents the graph.

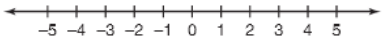
**a.** $x < 2$ or $x > 3$



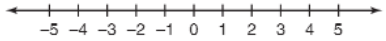
**b.** $-1 \geq x \geq -1$



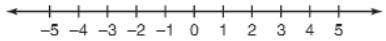

**c.** $x < 0$ or $x < 2$



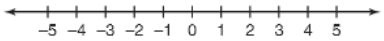

**d.** $x > 1$ and $x < -2$



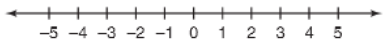

**e.** $x < 3$ and $x > 2$




**f.** $x < 2$ and $x < -1$




**g.** $x > -1$ or $x < 0$




To solve a compound inequality written in compact form, isolate the variable between the two inequality signs, and then graph the resulting statement. To solve an inequality involving "or," simply solve each inequality separately, keeping the word "or" between them, and then graph the resulting statements.

**4.** Solve and graph each compound inequality showing the steps you performed. Then, write the final solution that represents the graph.

**a.** $6 < x - 6 \leq 9$

**b.** $-2 < -x < 6$

**c.** $-4 \leq -3x + 1 \leq 12$

**d.** $2x + 7 < 10$ or $-2x + 7 > 10$

e. $\frac{1}{2}x + 3 > 4$ or $-x < 3$

2

f. $1 + 6x > 11$ or $x - 4 < -5$

Be prepared to share your solutions and methods.